

# Fast Finite Element Analysis of IPM Motors Using Block Model Order Reduction

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This paper presents fast finite element analysis of IPM motors using model order reduction (MOR) based on the proper orthogonal decomposition (POD). It is known that one needs long computational time for POD-based MOR applied to moving objects for which many basis vectors are necessary. In the present block MOR, the domain is subdivided into several blocks in each of which the basis vectors are constructed from snapshot solutions. The computational time of block MOR is shorter than that of the conventional MOR while accuracy of both methods are almost the same.

**Index Terms**— Permanent magnet machines, finite element analysis, reduced order systems.

## I. INTRODUCTION

IN the design of the control and driving systems of motors, equivalent circuits and behavior models of motors are widely used rather than finite element (FE) models which have too heavy computational burden for dynamic simulations. The accuracy of the former two methods is, however, often unsatisfactory. For this reason, fast and accurate computational methods for motor analysis have been required.

In order to reduce the computational time in FE analysis, the model order reduction (MOR) based on the proper orthogonal decomposition (POD) has been proposed and successfully applied to analysis of stationary electromagnetic devices [1][2]. However, conventional POD-based MOR is not very effective for analysis of moving objects such as motors and actuators because it needs many basis vectors for MOR to accurately express the electromagnetic fields in moving objects which significantly change in time [3].

In this paper, we discuss the validity of the block MOR method for motor analysis which has been proposed for fast analysis of moving objects and shown to perform fast and accurate analysis of a vibration energy harvester [4].

We will compare the computational performance of block MOR applied to analysis of IPM motors with that of conventional POD-based MOR and FE analysis.

## II. NUMERICAL METHOD

The FE equation of 2D magnetostatic field in IPM motors is

$$\sum_j A_j \int_{\Omega} (\mathbf{v} \cdot \text{grad} N_j) \cdot (\text{grad} N_j) dS$$

$$= \int_{\Omega} N_i J dS + \int_{\Omega} \left( -M_y \frac{dN_i}{dx} + M_x \frac{dN_i}{dy} \right) dS \quad (1)$$

where  $A_j$ ,  $\mathbf{v}$ ,  $N_j$ ,  $J$ ,  $M$  are magnetic vector potential, magnetic reluctance, scalar interpolation equation, current density and magnetization. The Newton-Raphson method is applied to (1) to obtain

$$\frac{\partial \mathbf{g}}{\partial \mathbf{A}} \Delta \mathbf{A} = -\mathbf{g} \quad (2)$$

where  $\mathbf{A} \in \mathbb{R}^n$  and  $\mathbf{g} \in \mathbb{R}^n$  are the solution and residual vectors. We apply POD-based MOR to (2) in order to reduce the

number of the degree of freedom. To do so, we solve (2) at  $s$  sampling angles  $\theta_i$ ,  $i = 1, 2, \dots, s$ . Then, the data matrix  $\mathbf{X}$  is constructed

$$\mathbf{X} = [\mathbf{A}(\theta_1) \quad \mathbf{A}(\theta_2) \quad \dots \quad \mathbf{A}(\theta_s)] \quad (3)$$

The singular value decomposition applied to  $\mathbf{X}$  results in

$$\mathbf{X} = \mathbf{W} \Sigma \mathbf{V}^t = \sigma_1 \mathbf{w}_1 \mathbf{v}_1^t + \sigma_2 \mathbf{w}_2 \mathbf{v}_2^t + \dots + \sigma_s \mathbf{w}_s \mathbf{v}_s^t \quad (4)$$

where  $\sigma_i$  is  $i$ -th eigenvalue of  $\mathbf{X}$ ,  $1 \leq i \leq s$ , and  $\mathbf{w}_i$ ,  $\mathbf{v}_i$  are the eigenvectors of  $\mathbf{X}\mathbf{X}^t$ ,  $\mathbf{X}^t\mathbf{X}$ , respectively, where the number of snapshots  $s$  is set much smaller than  $n$ . The solution  $\mathbf{A}$  to (1) is then approximately expressed in a form  $\mathbf{A} = \mathbf{W}\mathbf{y}$  where  $\mathbf{y} \in \mathbb{R}^s$ . Thus (2) reduces to

$$\mathbf{W}^t \frac{\partial \mathbf{g}}{\partial \mathbf{A}} \mathbf{W} \Delta \mathbf{y} = -\mathbf{W}^t \mathbf{g} \quad (5)$$

Because  $s \ll n$ , (5) can be solved much faster than (2).

To reduce the computational time,  $s$  should be set as small as possible. However, the computational accuracy deteriorates when  $s$  is too small to accurately express the electromagnetic field which significantly changes due to motor rotation. It is, therefore, important how to effectively take the snapshots of the field for the MOR analysis. In the conventional POD-based MOR method, the snapshots are taken at equal rotational intervals. However, this leads to many snapshots necessary for satisfactory accuracy in the solution.

In the present method, to overcome this difficulty, we subdivide the domain into  $m$  blocks and generate  $\mathbf{W}_i$  in each block as shown in Fig. 1. The transformation matrices  $\mathbf{W}_i$  are generated depending on rotor angle. Because the changes in the electromagnetic field in each block would be relatively small in comparison with that in the whole domain, the number of the basis vectors can be suppressed.

## III. NUMERICAL RESULTS

The conventional-MOR and present block MOR methods are applied to analysis of the IPM motor shown in Fig. 2 [5]. The analysis conditions are summarized in TABLE I. The air gap and other domain are discretized with rectangular and triangular finite elements whose number of the node is 15267.

In the conventional MOR,  $s$  snapshots are taken at equal intervals. The error in magnetic induction is defined by

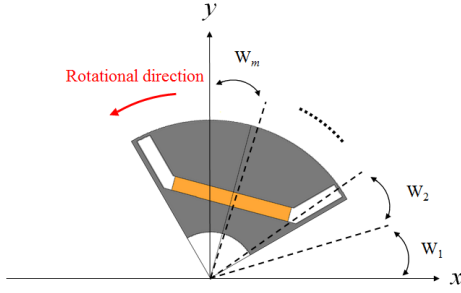


Fig. 1. Transformation matrices for  $m$  blocks.

$$\text{error}(\mathbf{B}) = \frac{1}{N_t} \sum_{n=1}^{N_t} \sqrt{\frac{\sum_{k=1}^{N_e} |\mathbf{B}_{k,n}^{\text{orig}} - \mathbf{B}_{k,n}^{\text{red}}|^2}{\sum_{k=1}^{N_e} |\mathbf{B}_{k,n}^{\text{orig}}|^2}} \quad (6)$$

where  $N_t$ ,  $N_e$ ,  $\mathbf{B}_{k,n}^{\text{orig}}$  and  $\mathbf{B}_{k,n}^{\text{red}}$  are the number of the time steps and finite elements, and the magnetic induction obtained by FEM and the conventional MOR, respectively. The errors and the computational time for the motor analysis to which conventional MOR is applied are summarized in TABLE II. Although the accuracy becomes better, with stagnation, as  $s$  increases, the computational time rapidly increases. This is mainly due to increase in computational burden for matrix-vector products in MOR process. This difficulty could be relaxed by using the discrete empirical interpolation [3]. We propose here an alternative remedy for this problem.

Figure 3 shows the computational times and numerical error of block MOR. It is found from Fig. 3 that the computational time reduces as the number of blocks  $m$  increases. This is because the number of snapshots in a block decreases as  $m$  increases. On the other hand, the computational error scarcely depends on  $m$ . Further increase in  $m$  is found to reduce computational time but lead to increase in error.

The error in torque  $T$  defined by

$$\text{error}(T) = \sqrt{\frac{\sum_{k=1}^{N_e} (T_k^{\text{orig}} - T_k^{\text{red}})^2}{\sum_{k=1}^{N_e} (T_k^{\text{orig}})^2}} \quad (7)$$

is comparatively shown for both MORs in Fig. 4. It is found from these results that the numerical errors again scarcely depend on  $m$ , and in addition their values are larger than those for the magnetic induction. This is because not only errors in magnetic induction but also in magnetic permeability attribute to the torque error.

#### IV. CONCLUSION

In this paper, we have discussed the performance of the conventional and block MOR methods which are applied to analysis of an IPM motor. It has been shown that the computational time in block MOR is shorter than that of the conventional MOR while there are no significant differences in the numerical accuracy of both methods. In the long version, we will discuss the performance of the present method for other computational settings and possibility to improve its numerical accuracy.

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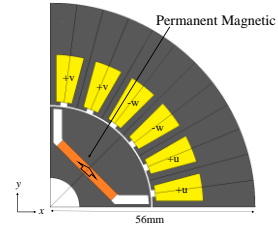


Fig. 2. IPM motor [5]

TABLE I  
ANALYSIS CONDITION

Parameter	Value
Current value (AT)	300
Phase angle of current (degree)	0
Magnetization (T)	1.4
Laminate thickness (mm)	65
Initial angle of rotor (degree)	15
Number of node	15267

TABLE II  
CONVENTIONAL ANALYSIS RESULTS OF MAGNETIC INDUCTION

$s$	$\text{error}(\mathbf{B})$	time
16	6.62%	19.1%
31	2.81%	45.6%
46	2.02%	$1.07 \times 10^{30}$
91	2.00%	$1.11 \times 10^{30}$

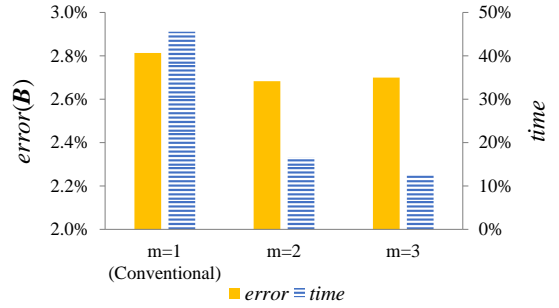


Fig. 3. Comparison of results between conventional and block MORs where snapshots are taken by  $3^\circ$  intervals.

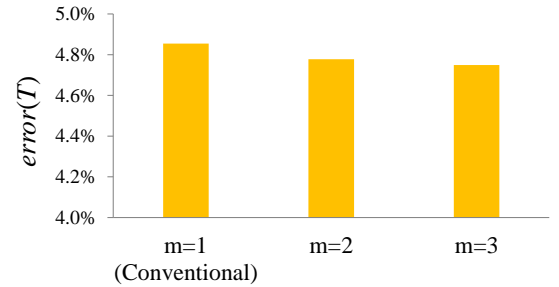


Fig. 4. Comparison of  $\text{error}(T)$  where snapshots are taken by  $3^\circ$  intervals.

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